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Tunneling Radiation of Charged and Magnetized Particles from the Reissner-Nordström-de Sitter Black Holes with Magnetic Charges

Liang Zhou · Jingyi Zhang

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Abstract We extended the Parikh-Wilczek's method to calculate the tunneling radiation of charged and magnetized particles from the event horizon and the cosmological horizon of the Reissner-Nordström-de Sitter black hole with magnetic charges. We reconstructed the electromagnetic field tensor and recalculated the Lagrangian of the field corresponding to the source with electric and magnetic charges. By viewing the eclectic and magnetic charges as an equivalent electric charge, we obtained the tunneling rate of the charged and magnetized particles. Our calculation supports the conclusion given by Parikh and Wilczek that the emission spectrum is no longer purely thermal, and the emission process supports the information conservation.

Keywords Black hole tunneling · Hawking radiation · Quantum theory

1 Introduction

In 1974, Stephen Hawking found that black holes can emit particles and the radiant spectrum is purely thermal [1, 2]. It caused a serious problem: information will be lost during the process of emission. In 2000, Parikh and Wilczek proposed a semi-classical method to solve this problem [3–5]. They treated the Hawking radiation as a tunneling process. In their tunneling framework, the energy conservation is enforced, and if we consider the energy conservation, we should take into account the back reaction on the geometry of the black hole. Then, the tunneling potential barrier is created just by the tunneling particle itself and. In this way, the emission spectrums of the particles from the spherically symmetric black holes, such as Schwarzschild black hole and Reissner-Nordström black hole, are calculated. The results are that the corrected spectrum deviates the pure thermal spectrum but satisfies an underlying unitary theory. Following their method, a number of static or stationary rotating black holes have been studied [6–18]. Recently, Parikh-Wilczek's semi-classical method was extended to calculate the tunneling rates of massive particles, charged particles and even the

L. Zhou · J. Zhang (✉)
Centre for Astrophysics, Guangzhou University, Guangzhou, 510006, China
e-mail: physicz@tom.com

charged and magnetized particles [19–38]. All the obtained results supports the results given by Parikh and Wilczek, that is, the emission spectrum is no longer purely thermal, and the emission process supports the information conservation.

In this paper, we continue to extend this method to calculate the emission rate of charged and magnetized particles from the Reissner-Nordström-de Sitter black holes.

2 Maxwell Equation Corresponding to the Source with Electric and Magnetic Charges

Following the method in the literatures [34–36, 39, 40], we first investigate the Maxwell equation and the Lagrangian of the electromagnetic field corresponding to the source with electric and magnetic charges.

For a source with electric and magnetic charges, the electromagnetic tensor is defined as

$$F_{\mu\nu} = \nabla_\nu A_\mu - \nabla_\mu A_\nu + G^+_{\mu\nu}, \tag{1}$$

where $G^+_{\mu\nu}$ is the Dirac string term. Then, the Maxwell equation can be written as

$$\nabla_\nu F^{\mu\nu} = 4\pi\rho_e u^\mu, \tag{2}$$

$$\nabla_\nu F^{+\mu\nu} = 4\pi\rho_g u^\mu, \tag{3}$$

where $F^{+\mu\nu}$ is the dual tensor of $F^{\mu\nu}$, ρ_e and ρ_g are the densities of electric and magnetic charges, respectively, while u^μ is the 4-velocity. For the sake of simplicity, we define a new real anti-symmetric tensor

$$\tilde{F}^{\mu\nu} = F^{\mu\nu}\cos\alpha + F^{+\mu\nu}\sin\alpha, \tag{4}$$

where α is a real constant angle. From (2) and (3) we have

$$\nabla_\nu \tilde{F}^{\mu\nu} = 4\pi(\rho_e\cos\alpha + \rho_g\sin\alpha)u^\mu, \tag{5}$$

$$\nabla_\nu \tilde{F}^{+\mu\nu} = 4\pi(-\rho_e\sin\alpha + \rho_g\cos\alpha)u^\mu. \tag{6}$$

If we let

$$\rho_e\cos\alpha + \rho_g\sin\alpha = \rho_h, \tag{7}$$

$$-\rho_e\sin\alpha + \rho_g\cos\alpha = 0, \tag{8}$$

that is, let $\rho_e/\rho_g = \cot\alpha$, then the Maxwell equation can be rewritten as

$$\nabla_\nu \tilde{F}^{\mu\nu} = 4\pi\rho_h u^\mu, \tag{9}$$

$$\nabla_\nu \tilde{F}^{+\mu\nu} = 0. \tag{10}$$

That is,

$$\frac{\partial}{\partial x^\nu}\left(\sqrt{-g}\,\tilde{F}^{\mu\nu}\right) = 4\pi\sqrt{-g}\,J^\mu, \tag{11}$$

where $J^\mu = \rho_h u^\mu$. Obviously, (9)–(11) are similar to the Maxwell equation corresponding to the source only with electric charges. In our discussion, we only discuss the tunneling of charged and magnetized particles. According to the no-hair theorem, most of the information about the black hole are lost. Without loss of generality, we can take the black hole as a conducing sphere and assume that the densities of electric and magnetic charges satisfy (7) and (8), that is, $\rho_e/\rho_g = \cot\alpha$, then we have

$$Q_h^2 = Q_e^2 + Q_g^2, \tag{12}$$

where Q_e and Q_g are the electric and magnetic charges, respectively, while Q_h is the equivalent charge corresponding to the density ρ_h. Similarly, we can construct the Lagrangian density of the electromagnetic field as follows

$$L_h = -\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}. \tag{13}$$

And the corresponding generalized coordinates are

$$\tilde{A}_\mu = (\tilde{A}_t, \tilde{A}_1, \tilde{A}_2, \tilde{A}_3), \tag{14}$$

which satisfies

$$\tilde{F}_{\mu\nu} = \nabla_\nu \tilde{A}_\mu - \nabla_\mu \tilde{A}_\nu. \tag{15}$$

3 Painlevé Coordinates and the Radial Motion Equation of Charged and Magnetized Particles

In Painlevé coordinates, the metric of the Reissner-Nordström-de Sitter black hole with magnetic charges has the form

$$\begin{aligned} ds^2 = &-\left(1 - \frac{2M}{r} + \frac{Q_e^2 + Q_g^2}{r^2} - \frac{\lambda}{3}r^2\right)dt^2 \\ &\pm 2\sqrt{\frac{2M}{r} - \frac{Q_e^2 + Q_g^2}{r^2} + \frac{\lambda}{3}r^2}\,dt\,dr + dr^2 + r^2 d\Omega^2, \end{aligned} \tag{16}$$

where λ is the cosmological constant, while the positive (negative) sign denotes the metric of the outgoing (ingoing) particle at the EH (CH). Substituting (12) into (16), the metric (16) becomes

$$\begin{aligned} ds^2 = &-\left(1 - \frac{2M}{r} + \frac{Q_h^2}{r^2} - \frac{\lambda}{3}r^2\right)dt^2 \\ &\pm 2\sqrt{\frac{2M}{r} - \frac{Q_h^2}{r^2} + \frac{\lambda}{3}r^2}\,dt\,dr + dr^2 + r^2 d\Omega^2. \end{aligned} \tag{17}$$

From the equation

$$1 - \frac{2M}{r} + \frac{Q_h^2}{r^2} - \frac{\lambda}{3}r^2 = 0, \tag{18}$$

we can get the inner horizon (IH), the event horizon (EH) and the cosmological horizon (CH) of the black hole, respectively, namely,

$$r_- = -\sqrt{Z_1} + \sqrt{Z_2} + \sqrt{Z_3}, \tag{19}$$

$$r_h = \sqrt{Z_1} - \sqrt{Z_2} + \sqrt{Z_3}, \tag{20}$$

$$r_c = \sqrt{Z_1} + \sqrt{Z_2} - \sqrt{Z_3}, \tag{21}$$

where $\sqrt{Z_1}$, $\sqrt{Z_2}$, $\sqrt{Z_3}$ and α satisfy

$$\sqrt{Z_1} = \frac{1}{\sqrt{2\lambda}}\left[1 + \sqrt{1 - 4\lambda Q_h^2}\cos\frac{\alpha}{3}\right]^{1/2}, \tag{22}$$

$$\sqrt{Z_2} = \frac{1}{\sqrt{2\lambda}}\left[1 - \sqrt{1 - 4\lambda Q_h^2}\cos\left(\frac{\alpha}{3} + \frac{\pi}{3}\right)\right]^{1/2}, \tag{23}$$

$$\sqrt{Z_3} = \frac{1}{\sqrt{2\lambda}}\left[1 - \sqrt{1 - 4\lambda Q_h^2}\cos\left(\frac{\alpha}{3} - \frac{\pi}{3}\right)\right]^{1/2}, \tag{24}$$

$$\alpha = \arccos\left[-\frac{1 - 18M^2\lambda + 12Q_h\lambda}{(1 - 4\lambda Q_h^2)^{3/2}}\right]. \tag{25}$$

Now, let us calculate the radial motion equation of charged and magnetized particles. As mentioned above, we treat the charged and magnetic particle as an equivalent electric charge. Similar to [34], the tunneling particle can be treated as a de Broglie wave and the expression of $\dot{r}$ is

$$\dot{r} = v_p = \frac{1}{2}v_g = -\frac{1}{2}\frac{g_{00}}{g_{01}} = \pm\frac{1}{2}\frac{r^2 - 2Mr + Q_h^2 - \frac{\lambda}{3}r^4}{\sqrt{2Mr^3 - Q_h^2 r^2 + \frac{\lambda}{3}r^6}}. \tag{26}$$

Here, the plus sign corresponds to the motion equation of the outgoing particle near the event horizon, and the minus sign corresponds to that of the ingoing particle near the cosmological horizon.

4 Tunneling from the Event Horizon

Let us first calculate the emission rate of the particle tunneling across the event horizon. When we take into account the self-gravitation of the tunneling particle with energy ω and the equivalent charge q_h, the effective line element and the expression of $\dot{r}$ become, respectively,

$$ds^2 = -\left(1 - \frac{2(M-\omega)}{r} + \frac{(Q_h - q_h)^2}{r^2} - \frac{\lambda}{3}r^2\right)dt^2 + 2\sqrt{\frac{2(M-\omega)}{r} - \frac{(Q_h - q_h)^2}{r^2} + \frac{\lambda}{3}r^2}\,dt\,dr + dr^2 + r^2 d\Omega^2, \tag{27}$$

$$\dot{r} = \frac{1}{2}\frac{r^2 - (2M-\omega)r + (Q_h - q_h)^2 - \frac{\lambda}{3}r^4}{\sqrt{2(M-\omega)r^3 - (Q_h - q_h)^2 r^2 + \frac{\lambda}{3}r^6}}. \tag{28}$$

Moreover, when we investigate the particle's tunneling process, the matter-gravity system consists of the black hole and the electromagnetic field outside the black hole. So the Lagrangian of the matter-gravity system can be written as

$$L = L_m + L_h. \tag{29}$$

From (13) $L_h = -\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}$, in order to eliminate the freedom corresponding to $\tilde{A}_\mu$ in the generalized coordinates $\tilde{A}_\mu = (\tilde{A}_t, \tilde{A}_1, \tilde{A}_2, \tilde{A}_3)$, we should rewrite the imaginary part of the action as the form

$$\begin{aligned}\operatorname{Im} S &= \operatorname{Im}\int_{t_i}^{t_f}(L - P_{\tilde{A}_t}\dot{\tilde{A}}_t)dt\\ &= \operatorname{Im}\left\{\int_{r_f}^{r_i}\left[P_r - \frac{P_{\tilde{A}_t}\dot{\tilde{A}}_t}{\dot{r}}\right]dr\right\}\\ &= \operatorname{Im}\left\{\int_{r_f}^{r_i}\left[\int_{(0,0)}^{(P_r,P_{\tilde{A}_t})} dP_r' - \frac{\dot{\tilde{A}}_t}{\dot{r}}dP'_{\tilde{A}_t}\right]dr\right\}.\end{aligned} \tag{30}$$

Here, $P_{\tilde{A}_t}$ is the canonical momentum of the electromagnetic field conjugate to $\tilde{A}_t$. According to Hamilton's equations, we have

$$\dot{r} = \left.\frac{dH}{dP_r}\right|_{(r;\tilde{A}_t;P_{\tilde{A}_t})} = \frac{d(M-\omega)}{dP_r}, \tag{31}$$

$$\dot{\tilde{A}}_t = \left.\frac{dH}{dP_{\tilde{A}_t}}\right|_{(\tilde{A}_t;r,P_r)} = \frac{Q_h - q_h}{r}\frac{d(Q_h - q_h)}{dP_{\tilde{A}_t}}. \tag{32}$$

Considering the conservation of the energy and the equivalent electric charge, the mass M and the equivalent electric charge Q_h are fixed. Substituting (28), (31) and (32) into (30), we have

$$\begin{aligned}\operatorname{Im} s = \operatorname{Im}\Bigg\{&\int_{(M,Q_h)}^{(M-\omega,Q_h-q_h)}\int_{r_i}^{r_f}\frac{2\sqrt{2(M-\omega)r^3 - (Q_h - q_h)^2r^2 + \frac{\lambda}{3}r^6}}{r^2 - (2M-\omega)r + (Q_h-q_h)^2 - \frac{\lambda}{3}r^4}\\ &\times\left[d(M-\omega) - \frac{Q_h - q_h}{r}d(Q_h - q_h)\right]dr\Bigg\}.\end{aligned} \tag{33}$$

We find that $r = r_h$ is a single pole in (33). We can evaluate the integral by deforming the contour around the pole to ensure that positive energy solution decay in time. Finishing the integration, we obtain

$$\operatorname{Im} S = -\frac{\pi}{2}(r_{fh}^2 - r_{ih}^2) = -\frac{1}{2}\Delta S_{EH}, \tag{34}$$

where ΔS_{EH} is the change of Bekenstein-Hawking entropy of the black hole before and after the emission across the event horizon. So the tunneling rate from the EH is

$$\Gamma \propto \exp[-2\operatorname{Im} S] = e^{\Delta S_{EH}}, \tag{35}$$

which is consistent with an underlying unitary theory.

5 Tunneling from the Cosmological Horizon

When the tunneling takes place at the de Sitter cosmological horizon, the particle is found to tunnel into the CH. When we take into account the self-gravitation of the tunneling particle with energy ω and charge q_h, the effective metric and the expression of $\dot{r}$ become, respectively,

$$ds^2 = -\left(1 - \frac{2(M+\omega)}{r} + \frac{(Q_h+q_h)^2}{r^2} - \frac{\lambda}{3}r^2\right)dt^2 - 2\sqrt{\frac{2(M+\omega)}{r} - \frac{(Q_h+q_h)^2}{r^2} + \frac{\lambda}{3}r^2}\,dtdr + dr^2 + r^2 d\Omega^2, \tag{36}$$

$$\dot{r} = -\frac{1}{2}\frac{r^2 - (2M+\omega)r + (Q_h+q_h)^2 - \frac{\lambda}{3}r^4}{\sqrt{2(M+\omega)r^3 - (Q_h+q_h)^2 r^2 + \frac{\lambda}{3}r^6}}. \tag{37}$$

In the same way, according to Hamilton's equations, we have

$$\dot{r} = \left.\frac{dH}{dP_r}\right|_{(r;\tilde{A}_t;P_{\tilde{A}_t})} = -\frac{d(M+\omega)}{dP_r}, \tag{38}$$

$$\dot{\tilde{A}}_t = \left.\frac{dH}{dP_{\tilde{A}_t}}\right|_{(\tilde{A}_t;r,P_r)} = -\frac{Q_h+q_h}{r}\frac{d(Q_h+q_h)}{dP_{\tilde{A}_t}}. \tag{39}$$

When the charged and magnetized particles income from the cosmological horizon, the imaginary part of the action should be

$$\operatorname{Im} s = \operatorname{Im}\left\{\int_{(M,Q_h)}^{(M+\omega,Q_h+q_h)}\int_{r_i}^{r_f}\frac{2\sqrt{2(M+\omega)r^3 - (Q_h+q_h)^2r^2 + \frac{\lambda}{3}r^6}}{r^2 - (2M+\omega)r + (Q_h+q_h)^2 - \frac{\lambda}{3}r^4} \times \left[d(M+\omega) - \frac{Q_h+q_h}{r}d(Q_h+q_h)\right]dr\right\}. \tag{40}$$

In (40) $r = r_c$ is also a pole. By using the same treatment as above mentioned, we obtain

$$\operatorname{Im} S = -\frac{\pi}{2}(r_{fc}^2 - r_{ic}^2) = -\frac{1}{2}\Delta S_{CH}. \tag{41}$$

Here, ΔS_{CH} is the change of Bekenstein-Hawking entropy of the black hole before and after the emission across the cosmological horizon. So the tunneling rate from the CH is

$$\Gamma - \exp[-2\operatorname{Im} S] = e^{\Delta S_{CH}}. \tag{42}$$

It is also consistent with an underlying unitary theory.

6 Conclusion

We have calculated the emission rates of charged and magnetized particles tunneling across the event horizon and the cosmological horizon of the Reissner–Nordström-de Sitter black hole with magnetized charges. In our discussion, we first reconstruct the Maxwell equation and rewrite the Lagrangian of the electromagnetic field corresponding to the source with electric and magnetic charges. Then, by treating a charged and magnetized particle as an equivalent charged particle, we obtain the emission rate of the charged and magnetized particle. We find that the emission rates of the charged and magnetized particles also have the same functional form as that of previous literatures [37, 38], that is, the emission spectrum is no longer pure thermal, but consists with an underlying unitary theory.

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